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# DETERMINATION OF CHARACTERISTICS OF FATIGUE CRACKS GROWTH IN THE AXLES OF RAILWAY CARRIAGES

Providing damage resistance is the most important requirement presented to railway axles. This paper provides a detailed review of the characteristics of resistance to fatigue crack growth in axles made of EA4T steel (25CrMo4). This steel is typical for axles of high-speed rolling stock. Resistance to fatigue crack growth is defined by the nature of the Paris-Erdogan curve $da/dN=CK^m$, the parameters of which are the coefficient $C$ and the exponent $m$. Determination of these parameters is based on the results of fatigue tests with the samples or with full-scale axles having cracks. The analysis of numerous scientific papers has shown that for the specified steel there is a significant scatter in the values of these parameters. This scatter is caused by a number of factors, the main of which are: the difference in test objects (from small samples of the SE(B) type to full-scale rolling axles 190 mm in diameter); the difference in loading methods (tension-compression, flat bending, circular bending); the variation of mechanical properties of EA4T steel. In the paper the models of surface fatigue crack geometry are analyzed and semi-elliptical shape of crack front line is selected as the most acceptable. The significant scatter in the values of the parameters $m$ and $C$ prevents to establish reliably the residual life of axles with cracks, which is an important characteristic for practical forecasting of axle survivability. The technique for parameters $m$ and $C$ optimization in view of semi-elliptical shape of the crack front line has been developed. The relationship linking the parameters $m$ and $C$ has been established. The method for determining the residual life of the axle by the criterion of non-destruction is proposed. The dependence between the residual life and the value of the parameters of the Paris-Erdogan equation in the development of fatigue failure is established. The procedure for axles monitoring timeframes setting to prevent their fatigue failure is discussed.

***Keywords:*** parameters of Paris-Erdogan equation, fatigue crack, residual life of axle.

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# ВИЗНАЧЕННЯ ХАРАКТЕРИСТИК ЗРОСТАННЯ ВТОМНИХ ТРІЩИН В ОСЯХ ЗАЛІЗНИЧНИХ ВАГОНІВ

Забезпечення опору пошкодженням – найважливіша вимога, яка пред'являється до залізничних осей. У статті детально розглянуто характеристики стійкості до поширення втомної тріщини осей зі сталі EA4T (25CrMo4), які використовуються для швидкісного рухомого складу. Стійкість до розвитку втомних тріщин визначається характером кривої Періса-Ердогана $da/dN=CK^m$, параметрами якої є коефіцієнт $C$ і показник ступеню $m$. Ці параметри встановлюють за результатами випробувань на втому зразків або натурних осей з тріщинами. Аналіз численних публікацій показав, що для зазначеної сталі спостерігається значний розкид значень цих параметрів. Такий розкид зумовлений низкою факторів, основними з яких є: різниця в досліджуваних об'єктах (від невеликих зразків типу SE(B) до натурних катаних осей діаметром 190 мм); відмінність способів навантаження (розтяг-стиск, плоский вигин, круговий вигин); розкид механічних властивостей сталі EA4T. Проаналізовано відомі моделі геометрії поверхневих втомних тріщин та обрано найбільш прийнятну напівеліптичну форму їх лінії фронту. Значний розкид значень параметрів $m$ і $C$ не дозволяє достовірно встановити важливу для практичного прогнозування живучості характеристику – залишковий ресурс осей з тріщиною. Розроблено методику оптимізації цих параметрів, яка враховує напівеліптичну форму лінії фронту тріщини. Отримана формула, що зв'язує параметри $m$ і $C$. Запропоновано метод визначення залишкового ресурсу осей за критерієм неруйнування. Встановлено залежність залишкового ресурсу від величини параметрів рівняння Періса-Ердогана при розвитку втомного руйнування. Обговорюється порядок встановлення термінів контролю осей для запобігання їх втомному руйнуванню.

***Ключові слова***: параметри рівняння Періса-Ердогана, втомна тріщина, залишковий ресурс осей.

**Introduction**. Cyclic changes in loads, instability of operating conditions of machines, dispersion in mechanical properties of materials and in geometric characteristics of their parts in case of unfavorable combination lead to destruction. Under the cyclic loads the destruction has a fatigue character. For the parts experiencing the circular bending (axles in particular), fatigue cracks appear on their surface in the area of maximum stress σ action; they develop at an increasing rate determined by the ratio of the crack depth to the quantity of loading cycles $N$.

The crack arises at the point of the axle surface where the effective stress concentration factor $K_\sigma$ has its maximum. In cylindrical parts the front line of the fatigue crack is most often convex elliptical [1] and it is almost orthogonal to the contour of part surface, Fig. 1. Less often the front line of the fatigue crack is concave sickle-shaped (at high coefficient $K_\sigma$ values, which are not typical for axles and shafts).

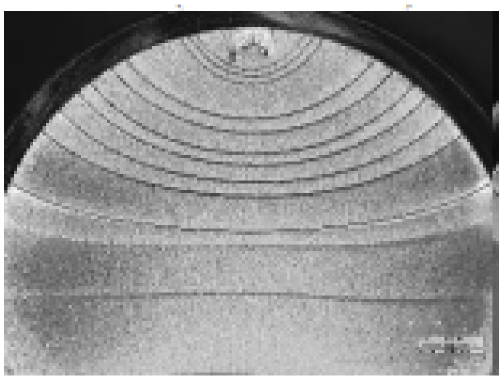

Fig. 1 – Fatigue crack front lines [1]

To provide the fatigue failure resistance the axles are made of medium carbon and low-alloy steels [2].

The process of fatigue crack development has three stages, Fig. 2. The second one (which typically is the longest of them) takes place at values of stress intensity factor (SIF) $K = \sigma\sqrt{\pi a}$ exceeding SIF threshold value $K_{th}$. Fatigue failure occurs when SIF reaches its critical value $K_{fc}$.

The crack growth rate (CGR) is described by well-known equations: Paris-Erdogan's equation, Walker's equation, NASGRO equation and others [3]. CGR modeling is also performed by numerical methods: method of finite elements and method of boundary elements [4, 5].

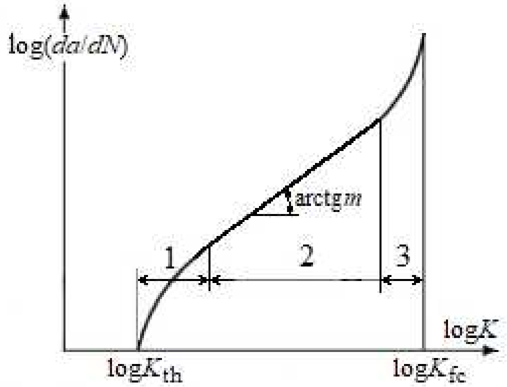


Fig. 2 – The scheme of kinematic diagram of fatigue failure

Paris-Erdogan's equation is simple and widely used to describe CGR in stage 2

$$\frac{da}{dN} = CK^m = C(\sigma\sqrt{\pi a})^m, \tag{1}$$

where coefficient $C$ and exponent $m$ are the parameters which are defined by the characteristics of the material for describing crack growth at stage 2.

The analysis of scientific papers shows that the value of these parameters for the same material often do not match. Therefore, it is impossible to calculate accurately the residual life of a part with a crack which is an important characteristic for practical prediction of survivability. This problem is typical for Paris-Erdogan's equation and other CGR equations.

**The purpose of this paper** is to develop the methodology for determining parameters $m$ and $C$ of formula (1) based on the results of measurements of a growing fatigue cracks in cylindrical parts. The technique is demonstrated on the example of axles made of EA4T steel (25CrMo4) using the results of fatigue tests given in well-known publications.

**Presentation of the main material.** Parameters $m$ and $C$ are determined by the results of fatigue tests of full-scale parts or specimens made from the same materials. For this purpose the integral of equation (1) is used

$$N_k = \frac{1}{C\sigma^m \pi^{0.5m}} \int_{ao}^{ak} a^{-0.5m} da = \frac{1}{C\sigma^m \pi^{0.5m}} \cdot \frac{a_k^{1-0.5m} - a_0^{1-0.5m}}{1-0.5m}, \tag{2}$$

where $a_0$ – the crack depth at $N$=0 (the beginning of fatigue test); $a_k$ – the crack depth after $N_k$ loading cycles.

If the depth $a_0$ and the stress for testing σ are given, and two pairs of values ($a_i$, $N_i$) and ($a_k$, $N_k$) are defined, then formula (2) forms a system of two transcendental equations, of which one can find the parameters $m$ and $C$ for equation (1). Usually there are much more than two such pairs of values, so the problem turns out to be over-determined and the parameters $m$ and $C$ are calculated by statistical methods.

During testing the length of the crack arc on the surface of the cylindrical sample $2s(N)$ is measured. Here $s = R\theta$ and the corresponding crack depth $a$ is setting indirectly, Fig. 3.

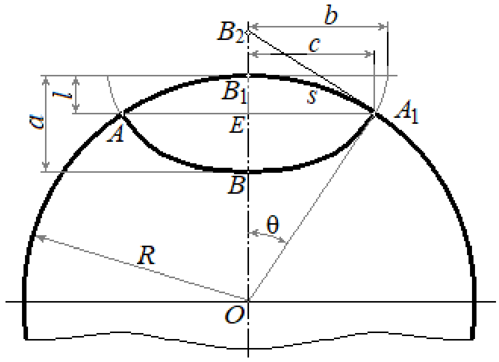


Fig. 3 – The geometric characteristics of the front line of a fatigue crack in cylindrical axle

The analysis of the traces of the front line of growing crack on the surface of destroyed object (they are called beach lines) shows that the curvature of these lines decreases, see Fig. 1. When an angle size is $\theta = \pi/2$, the curvature becomes $\kappa = 0$ (the front line becomes straight), and the crack depth is $a = R$. The front line of deeper cracks, with $a>R$, bends again, wherein the sign of the curvature becomes opposite. To calculate the values of $a$ on the measured values of $2s$, one of the dependencies satisfying the specified conditions is usually used:

$$a = c = K_f R \sin\theta, \quad K_f = 0.5 \div 1.0, \tag{3}$$

$$a = R\frac{\sin\theta + \cos\theta - 1}{\cos\theta}, \tag{4}$$

$$a = 2s/\pi = 2R\theta/\pi, \tag{5}$$

$$a = l = R(1-\cos\theta). \tag{6}$$

The graphs of dependencies (3) – (6) at $R$=100 mm are shown in Fig. 4.

The dependence (3) was proposed in the paper [6]. It satisfies the condition $a\big|_{\theta=\pi/2} = R$ at $K_f$=1 (curve 1), but is recommended for use at $K_f$=0.8 (curve 2) for the angles in the range of $\theta \le (1 \div 1.2)$ rad. The dependence (4) defines the shape of a fatigue crack front line $ABA_1$ as an arc of a circle with radius $B_2A_1 = Rtg\theta$, perpendicular to the contour of the part (curve 3) [7]. The dependence (5) defines the crack front line $ABA_1$ as an ellipse arc centered at the crack initiation point $B_1$, with the semi-axes dimensions $a$ and $b$ [8]. The cracks determined by the dependence (6) have a straight front line $AA_1$, their depth $a=l$ is the smallest of all reviewed (curve 5) [9, 10]. The intermediate position of curve 3 and the simplicity of determining it dependence (5) allow considering it to be acceptable for practical calculations of the values $a$ by fixed meanings $2s$.

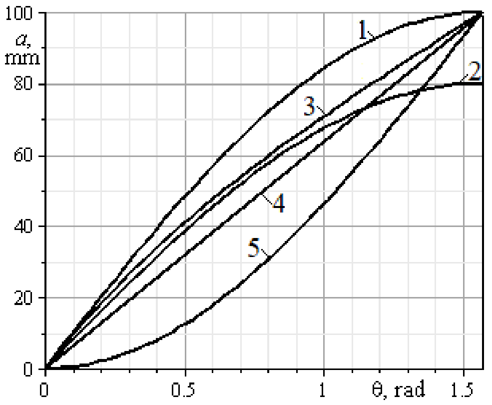


Fig. 4 – The graphs of analytical dependencies for the depth $a(\theta)$ of a growing crack at $0 \le \theta \le \pi/2$

In addition to visual determination of crack sizes, instrumental methods are also used: magnetic particle inspection, method of electric current potential reduction, ultrasonic diagnostics and acoustic emission [2]. The variety of methods for determining the crack depth $a$, the use of samples of different shapes and sizes, the dispersion of mechanical properties of axles material cause a significant spread in the values of the parameters $m$ and $C$, established by different authors.

The analysis of parameters $m$ and $C$ is performed for the axle steel EA4T (standard DSTU EN 13261:2018). Its chemical composition and mechanical properties are given in Tables 1 and 2.

Table 1 – Chemical composition of steel EA4T (25CrMo4) (the rest is Fe)

| C, % | Si, % | Mn, % | Ni, % |
|---|---|---|---|
| 0.22÷0.29 | 0.15÷0.40 | 0.5÷0.9 | 0.3 |
| S, % | P, % | Cr, % | Cu, % |
| 0.015 | 0.02 | 0.9÷1.2 | 0.3 |

Table 2 – Mechanical properties of steel EA4T

| Ultimate strength $\sigma_u$, MPa | Yield strength $\sigma_{0.2}$, MPa | Elongation at break δ, %. |
|---|---|---|
| 650÷800 | 420 | 18 |

In Table 3 the parameters of the dependence (1) known after the published data are established for the value of asymmetry coefficient of loading cycle $R_\sigma = -1$.

It is known that the parameters $m$ and $C$ depend on each other and this dependence can be presented in a linearized form

$$\log C = A + Bm. \tag{7}$$

Statistical analysis of the values given in table 3 gives us the regression line equation (7) in the form

$$\log C = -2.003 - 3.658m. \tag{8}$$

with correlation coefficient $r = -0.703$. In Table 3 the quantity of data is insignificant, $n$=7, therefore, it is undesirable to recommend the available limited sample of parameters $m$ and $C$ for practical calculations.

Table 3 – Parameters of the Paris-Erdogan`s equation for steel EA4T after the publiched data

| Sources | Objects | $m$ | $C$ |
|---|---|---|---|
| [6] | 6×40 | 3.25 | $6.68 \cdot 10^{-15}$ |
| [9] | ø190 | 3.2 | $2{,}70 \cdot 10^{-14}$ |
| | ø195/65 | 3.24 | $2.8 \cdot 10^{-14}$ |
| [11] | ø50 | 3.5 | $4.61 \cdot 10^{-15}$ |
| [12] | 10×24 | 3.2 | $4.22 \cdot 10^{-15}$ |
| [13] | 10×24 | 3.2 | $4.34 \cdot 10^{-15}$ |
| [14] | 10×24 | 3 | $5.21 \cdot 10^{-13}$ |

To supplement Table 3 it is proposed to use the results of experiments of a number of authors who obtained dependence (1) in graphical form. An example of such a representation is provided in Fig. 5.

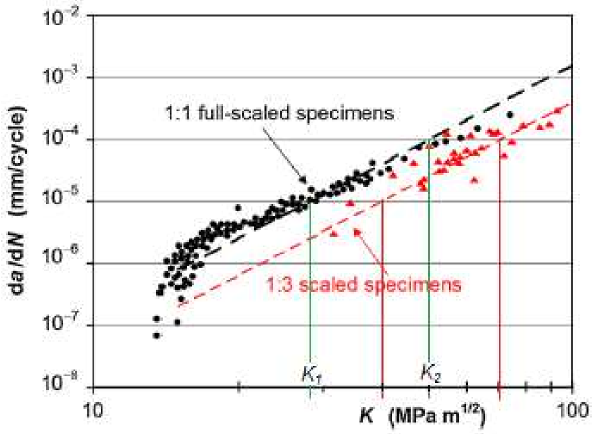


Fig. 5 – Kinematic fatigue fracture diagrams for specimens of steel EA4T according to [11]

To calculate $m$ and $C$ basing on these graphs, the logarithm of dependence (1) is used in the form

$$\log C + m \log K = \log(da/dN), \tag{9}$$

where $m$ and $\log C$ – the desired quantities.

If two arbitrary points are chosen on the rectilinear section of the graph «1:1 full-scaled specimens» (Fg. 5), then a system of two equations can be formed, of which $m$ and $\log C$ are calculated:

$$m = \frac{\log(da/dN)_2 - \log(da/dN)_1}{\log K_2 - \log K_1}, \tag{10}$$

$$\log C = \log(da/dN)_1 - \log K_1. \tag{11}$$

The coordinates of two selected points of the graph «1:1 full-scaled specimens» are $K_1 = 28.25\ \text{MPa}\sqrt{\text{m}}$, $\log(da/dN)_1 = -5$ and $K_2 = 50\ \text{MPa}\sqrt{\text{m}}$, $\log(da/dN)_2 = -4$, Fig. 5.

For these coordinates calculations by the formulas (10) and (11) produce the parameter values $m$=4.049 and

$$C = 1.32 \cdot 10^{-11} \frac{\text{mm}}{\left(\text{MPa}\sqrt{\text{m}}\right)^m} = 1.12 \cdot 10^{-17} \frac{\text{mm}}{\left(\text{MPa}\sqrt{\text{mm}}\right)^m}.$$

The similar calculations for the graph «1:3 scaled specimens» produce the parameter values $m$=4.040 and

$$C = 3.37 \cdot 10^{-12} \frac{\text{mm}}{\left(\text{MPa}\sqrt{\text{m}}\right)^m} = 2.93 \cdot 10^{-18} \frac{\text{mm}}{\left(\text{MPa}\sqrt{\text{mm}}\right)^m}.$$

The results of the similar calculations carried out by the formulas (10) and (11) for kinematic diagrams taken from published literature are given in Table 4.

Table 4 – Parameters $m$ and $C$, obtained by recalculation from graphs of $da/dN$ as functions of SIF's

| Sources | Objects | $m$ | $C$ |
|---|---|---|---|
| [11] | ø165 | 4.049 | $1.119 \cdot 10^{-17}$ |
| | ø55 | 4.040 | $2.927 \cdot 10^{-18}$ |
| [12] | 10×24 | 3.196 | $4.21 \cdot 10^{-15}$ |
| [15] | 10×24 | 3.1235 | $7.939 \cdot 10^{-15}$ |
| | ø180 | 3.753 | $3.074 \cdot 10^{-17}$ |
| [16] | 6×50 | 3.543 | $3.1795 \cdot 10^{-17}$ |
| [17] | 6×20 | 3.149 | $5.3986 \cdot 10^{-15}$ |
| [18] | 5×60 | 2.943 | $2.3776 \cdot 10^{-15}$ |
| [19] | ø176 | 3.303 | $2.6950 \cdot 10^{-15}$ |
| | ø176 | 4.052 | $2.2458 \cdot 10^{-17}$ |
| | ø176 | 3.298 | $3.3479 \cdot 10^{-15}$ |
| | ø176 | 3.126 | $6.2244 \cdot 10^{-15}$ |
| [20] | 50×20 | 3.325 | $1.8146 \cdot 10^{-15}$ |
| [21] | ø180 | 3.4469 | $9.1858 \cdot 10^{-15}$ |
| | ø180 | 2.9416 | $9.2244 \cdot 10^{-15}$ |
| | ø55 | 3.0711 | $9.1858 \cdot 10^{-15}$ |
| | 6×50 | 2.8342 | $3.2296 \cdot 10^{-14}$ |

Statistical analysis of $m$ and $C$ values, presented in Tables 3 and 4, provides for these parameters of steel EA4T the equation of regression line

$$\log C = -4.154 - 3.159m \tag{12}$$

with a sufficiently high correlation coefficient $r = -0.868$.

The values of $m$ and log$C$ were obtained from the referred papers, as well as the regression graphs plotted for them, Fig. 6.

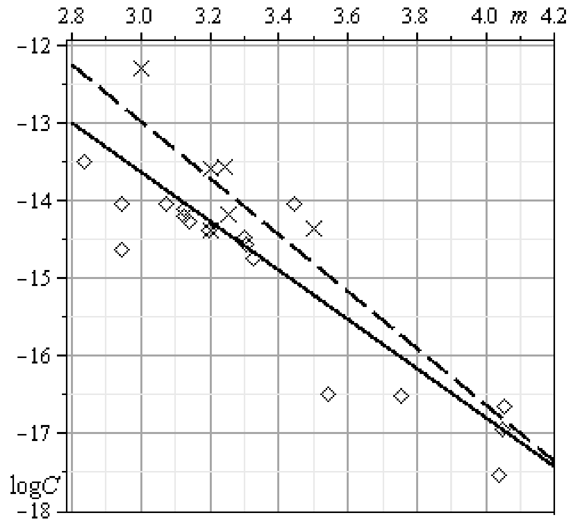


Fig 6 – Dependence of log$C$ on $m$: × – data in Table 3 and their regression line (dashed); ◇ – data in Table 4, and the regression line according to Tables 3 and 4 (Solid)

The increase in the number of observations led to the increase by 23.5 % in the reliability of estimating the values of the parameters in dependence (1) by the correlation coefficient.

To determine the parameters of equation (1), the dependence (12) is not sufficient, since the defining parameter $m$ must be known to use it. To find the latter, it is proposed to use the experimentally established values of the crack depth $a_i$ with the appropriate number of loading cycles $N_i$. These data allows us to determine the optimal combination of parameters $m$ and $C$.

As an example of such a calculation the experimental data presented in [9] are use. They are obtained at the stress $\sigma = 160\,\text{MPa}$ for the axle $D$=190 mm in diameter. When describing these results, it was assumed that a crack with a size of 2$s$ has a depth of $a = l$, see Fig. 3. Numerous observations on fatigue failures of axles indicate that their crack front lines are predominantly convex and therefore have a depth of $a >> l$. On this basis, according to the values of $l$ given in work [9], the depth of a crack with an elliptical front was determined using the joint solution of equations (5) and (6):

$$a = \frac{2R}{\pi} \arccos\left(1 - \frac{l}{R}\right). \tag{13}$$

The results of the performed calculations are given in Table 5.

Table 5 – Calculated crack depths for axle steel EA4T

| Number of measurements | Data from the paper [9] | | Calculations on (13) |
|---|---|---|---|
| | $N$, cycles | $l$, mm | $a$, mm |
| | 0 | 8.28 | 23.2 |
| 1 | 16 860 | 9.65 | 25.4 |
| 2 | 33 715 | 11.6 | 27.5 |
| 3 | 56 570 | 15.7 | 30.2 |
| 4 | 78 860 | 17.4 | 35.3 |
| 5 | 101 140 | 18.8 | 37.2 |
| 6 | 120 860 | 23.7 | 38.8 |
| 7 | 135 145 | 26.8 | 43.7 |
| 8 | 140 860 | 27.0 | 46.6 |
| 9 | 146 285 | 31.2 | 46.8 |
| 10 | 146 570 | 34.9 | 50.5 |
| 11 | 157 715 | 39.2 | 53.6 |
| 12 | 168 860 | 48.7 | 57.0 |
| 13 | 180 000 | 57.9 | 64.2 |
| 14 | 186 860 | 64.6 | 70.7 |
| 15 | 192 570 | 77.7 | 75.3 |
| 16 | 198 290 | | 83.9 |

The following algorithm is proposed for calculating parameters $m$ and $C$ of equation (1) based on the experimental data.

Step 1: The defining parameter is varied starting from the initial value ($m$ = 2.4 for example) with a step $\Delta m = 0.2$.

Step 2: At each step the parameter $C$ is determined by the least squares method [9]

$$C = \frac{\sum_{i=1}^{i=n}\left(a_i^p - a_0^p\right)^2}{p\cdot\sigma^m\cdot\pi^{0.5m}\cdot\sum_{i=1}^{i=n}N_i\left(a_i^p - a_0^p\right)}, \tag{14}$$

where $p = 1 - 0.5m$ – the exponent;

$n$ – the number of measurements;

$\sigma$ – the test stress.

Step 3: According to the values of $m$, $C$ and $N_i$, the calculated respective crack depths are determined from the integral of equation (1)

$$a_{ci} = \left(a_0^p + C\cdot p\cdot\sigma^m\cdot\pi^{0.5m}\cdot N_i\right)^{1/p}, \tag{15}$$

where $a_0$ – the depth of the initial fatigue crack (at $N = 0$).

Step 4: The variance of the quantity is determined $(a_{ci} - a_i)$ by the dependence

$$\mathrm{Var}(a_{ci} - a_i) = \frac{1}{n}\sum_{i=1}^{i=n}\left[(a_{ci} - a_i) - \bar{a}\right]^2, \tag{16}$$

where $\bar{a} = \frac{1}{n}\sum_{i=1}^{i=n}(a_{ci} - a_i)$ – the mean value of the difference $(a_{ci} - a_i)$.

Step 5: The next value $m + \Delta m = 0.2$ is taken.

This procedure is repeated until the minimum variance value (16) is reached, at which the combination of the parameters of equation (1) is optimal. To increase the accuracy of calculations, the step $\Delta m$ should be reduced.

Determination of parameters $m$ and $C$ for axles of steel EA4T is exemplified by the calculation for $a_i$ and $N_i$ values given in Table 5. The graph of variance (16) calculation with varying parameter $m$ is shown in Fig. 7.

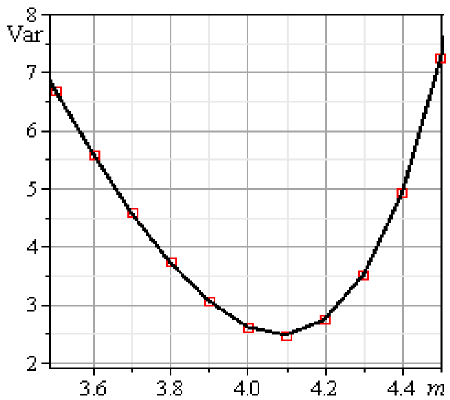


Fig. 7 – Determining the value of the parameter $m$ by minimum variance (16)

The minimum variance (16) with its value 2.38 corresponds to parameter $m$ value $m$ = 4.1. The second parameter of equation (1) is calculated using formula (14), its value is $C = 1.144\cdot 10^{-17}$.

The found values of $m$ and $C$ correspond well with the values of these parameters obtained in paper [7] for full-size axles, Table 4.

Graphs of the dependences $a(N)$ (15) were obtained: they prove that found parameters optimally describe the process of crack growth, Fig. 8. It is shown that at $m$ = 3.2 (variance $\mathrm{Var}(a_{ci} - a_i) = 10.4$) and at $m$ = 4.8 (variance $\mathrm{Var}(a_{ci} - a_i) = 23.2$) the discrepancy with the experimental results increases.

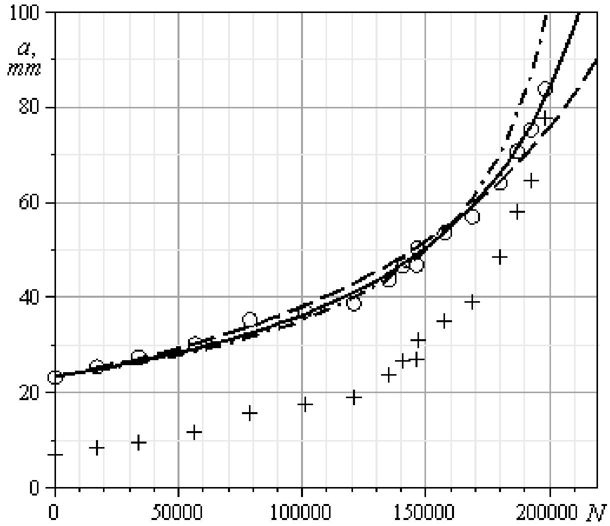


Fig. 8 – Growth of the fatigue crack depth $a(N)$ in the axle of EA4T steel: + – results of the experiment under the assumption of a rectilinear front line [9]; ○ – recalculation of the results [9] for elliptical front line. Curves using the dependence (14) at: $m$=3.2 (▬ ▬); $m$=4.1 (▬▬); $m$=4.8 (▬ • ▬)

For full-size axles with diameter $D \geq 160$ mm (Table 4) and the parameters $m$ and $C$ obtained here for $D = 190$ mm, the equation of the regression line was found

$$\log C = -5.357 - 2.790m, \tag{17}$$

with the correlation coefficient $r = -0.951$. That shows a significant correlation between the parameters $m$ and $\log C$.

In determining the residual life of railway axles with diameter $D > 160$ mm made of the considered steel EA4T, the exponent $m$ should be assigned and the coefficient $C$ should be found from formula (17). If the stress $\sigma$ that arises in axle operation is known and a surface crack with its depth $a_0$ is detected, then the residual life of the axle can be assessed using the formula obtained from dependence (15)

$$N_{res} = \frac{a_{\lim}^p - a_0^p}{C\cdot p\cdot\sigma^m\cdot\pi^{0.5m}}, \tag{18}$$

where $a_{\lim}$ is the limiting value of the fatigue crack depth at which stage 3 of catastrophic failure begins, see Fig. 2.

Cracks with a depth $a_0$ less than 2 mm cannot be detected by non-destructive testing methods (for example, by ultrasonic flaw detection). Therefore, to calculate the residual life of axles with shallow cracks, it is recommended to assign $a_0$=2 mm. As a limiting value for the depth of a fatigue crack, we can recommend

$a_{\lim} = D/3$. That corresponds to the size on the surface $2s = \pi D/3$, see dependence (5).

Estimation of the residual life value is exemplified with an axles 190 mm in diameter ($D$ = 190 mm).

In connection with significant scatter in experimentally determined value of parameter $m$ (from 2.9 to 4.1, table 4), the calculation is performed for the most typical values from this range: $m$=3.0; 3.5; 4.1. Chosen parameter $m$ values correspond to the following values of the second parameter: $C$=1.875·10$^{-14}$; 7.551·10$^{-16}$; 1.600·10$^{-17}$, see dependence (17). Crack depth values: initial $a_0$=2 mm, maximum $a_{\lim} = D/3 = 63.33$ mm. The stress in dependence (18) varies from 100 to 170 MPa. The results of the calculation by dependence (18) are presented in Fig. 9.

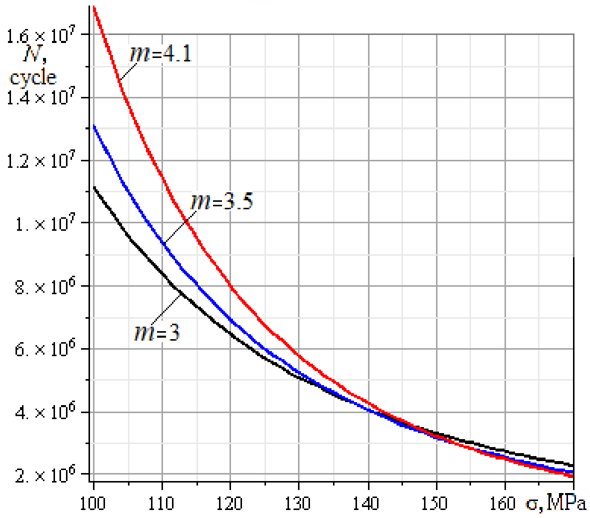


Fig. 9 – The influence of parameter $m$ value on the level of residual resource

The calculation shows that with an increase in the parameter $m$, there is an inverse relationship between the value of the residual resource and the level of acting stresses. Currently, railway axles are operated at stresses $\sigma = (140...160)$ MPa. For this stress range, change in parameter $m$ has insignificant effect on the value $N_{res}$, Fig. 9.

If at a given stress level σ the value $N_{res}$ is calculated, then to prevent destruction of the axle in further operation, the deadline for the next ultrasonic check can be recommended. For example, the number of loading cycles before such a control is recommended to be set $N_T = N_{res}/S$, with the safety factor $S$=1.2÷1.5. It should be borne in mind that the presence of a crack, as a rule, is not allowed, and if a crack is detected, the axle must be repaired or replaced.

## 3. Conclusions

1. The analysis of coefficients $m$ and $C$ of Paris-Erdogan`s equation is performed on the basis of published experimental data for EA4T (25CrMo4) steel, which is used for the manufacture of axles. The variety of sample designs and test equipment causes a significant scatter in the value of these parameters and prevents to indicate definitely their meanings for practical calculations.

2. The methodology for calculating the coefficients of the Paris-Erdogan`s equation is proposed; it is based on minimizing the variance of $(a_{ci} - a_i)$ values. This methodology made it possible to obtain the minimum deviation of the line $a(N)$ from the experimentally established coordinates of the points of fatigue crack front depth $a_i$ and corresponding quantities of loading cycles $N_i$.

3. The method for calculating the residual life of axle according to the criterion of non-destruction is presented. The calculation shows that at low stress levels, $\sigma < 120$ MPa the value of the residual life increases significantly with increasing parameter $m$. It is shown that for real stress level in the axles, $\sigma = (140...160)$ MPa the parameter $m$ values given in the references practically do not effect on the value of residual life.

4. Verification of proposed methodology is carried out.

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